

實踐大學 資訊科技與通訊學系			課程名稱	線性代數	
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(1) 用高斯喬丹法

$$3x_1 - x_2 + x_3 + 5x_4 = 3$$

$$-2x_1 - x_2 - x_3 - 3x_4 = 9$$

$$-2x_1 + x_2 - 7x_4 = 8$$

$$\begin{array}{ccc|ccc} 3 & -1 & 1 & 5 & 3 & 0+(-9)+(-24)-2-(-5) \\ -2 & -1 & -1 & -3 & 9 & = -31-2+14 \\ -2 & 1 & 0 & -7 & 8 & = -19 \end{array}$$

$$1 \begin{bmatrix} 3 & -1 & 1 & 5 & 3 \\ -2 & -1 & -1 & -3 & 9 \\ -2 & 1 & 0 & -7 & 8 \end{bmatrix}$$

$$4 \begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{5}{3} & 1 \\ 0 & 1 & -1 & -1 & 33 \\ 0 & \frac{1}{3} & \frac{2}{3} & \frac{-11}{3} & 10 \end{bmatrix}$$

$$7 \begin{bmatrix} 1 & 0 & 0 & 2 & 12 \\ 0 & 1 & 0 & -3 & 32 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

$$x_1 = 12 - 2t$$

$$x_2 = 32 + 3t$$

$$x_3 = -1 + 4t$$

$$x_4 = t$$

$$2 \begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{5}{3} & 1 \\ -2 & 1 & -1 & -3 & 9 \\ -2 & 1 & 0 & -7 & 8 \end{bmatrix}$$

$$5 \begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{5}{3} & 1 \\ 0 & 1 & -1 & -1 & 33 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

$$x_1 + 2x_4 = 12$$

$$x_2 - 3x_4 = 32$$

$$x_3 - 4x_4 = -1$$

$$x_4 = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 12 \\ 32 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

(2) 用反矩陣

$$\begin{bmatrix} 1 & 4 & 1 \\ -1 & -3 & -2 \\ 2 & 6 & 6 \end{bmatrix} X = \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix}$$

$$1 \begin{bmatrix} 1 & 4 & 1 & | & 10 & 0 \\ -1 & -3 & -2 & | & 0 & 10 \\ 2 & 6 & 6 & | & 0 & 0 & 1 \end{bmatrix}$$

$$2 \begin{bmatrix} 0 & 1 & -1 & | & 1 & 10 \\ -1 & -3 & -2 & | & 0 & 10 \\ 2 & 6 & 6 & | & 0 & 0 & 1 \end{bmatrix}$$

$$3 \begin{bmatrix} 0 & 1 & -1 & | & 1 & 10 \\ -1 & -3 & -2 & | & 0 & 10 \\ 0 & 0 & 2 & | & 2 & 2 & 1 \end{bmatrix}$$

$$4 \begin{bmatrix} 0 & 1 & -1 & | & 1 & 10 \\ -1 & 0 & -5 & | & 3 & 4 & 5 \\ 0 & 0 & 2 & | & 0 & 5 & 2 \end{bmatrix}$$

$$5 \begin{bmatrix} 0 & 1 & -1 & | & 1 & 10 \\ -1 & 0 & 0 & | & 3 & 9 & \frac{5}{2} \\ 0 & 0 & 1 & | & 0 & 1 & \frac{1}{2} \end{bmatrix}$$

$$6 \begin{bmatrix} 1 & 0 & 0 & | & -3 & -9 & \frac{-5}{2} \\ 0 & 1 & 0 & | & 1 & 2 & \frac{1}{2} \\ 0 & 0 & 1 & | & 0 & 1 & \frac{1}{2} \end{bmatrix}$$

$$X = \begin{bmatrix} -3 & -9 & \frac{-5}{2} \\ 1 & 2 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} -95 \\ 23 \\ 9 \end{bmatrix}$$

(3) 用克拉姆法则

$$\begin{aligned} 3x_1 + 2x_2 + x_3 &= 5 \\ x_1 + 2x_2 - 2x_3 &= 5 \\ 6x_1 + 1x_2 + 6x_3 &= 5 \end{aligned}$$

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & -2 \\ 6 & 1 & 6 \end{vmatrix} \begin{vmatrix} 3 & 2 \\ 1 & 2 \\ 6 & 1 \end{vmatrix} \det(A) = 36 + (-24) + 1 - 12 - (-6) - 12 = 12 + 1 - 18 = -5$$

$$x_1 = \frac{\begin{vmatrix} 5 & 2 & 1 \\ 5 & 2 & -2 \\ 5 & 1 & 6 \end{vmatrix} \begin{vmatrix} 5 & 2 \\ 5 & 2 \\ 5 & 1 \end{vmatrix}}{\det(A)} = \frac{60 + (-20) + 5 - 10 - (-10) - 60}{-5} = \frac{-15}{-5} = 3$$

$$x_2 = \frac{\begin{vmatrix} 3 & 5 & 1 \\ 1 & 5 & -2 \\ 6 & 5 & 6 \end{vmatrix} \begin{vmatrix} 3 & 5 \\ 1 & 5 \\ 6 & 5 \end{vmatrix}}{\det(A)} = \frac{90 + (-60) + 5 - 30 - (-30) - 30}{-5} = \frac{5}{-5} = -1$$

$$x_3 = \frac{\begin{vmatrix} 3 & 2 & 5 \\ 1 & 2 & 5 \\ 6 & 1 & 5 \end{vmatrix} \begin{vmatrix} 3 & 2 \\ 1 & 2 \\ 6 & 1 \end{vmatrix}}{\det(A)} = \frac{30 + 60 + 5 - 60 - 15 - 10}{-5} = \frac{10}{-5} = -2$$

$$A = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix} \# \checkmark$$

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(4) 用 LU 角解

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 2 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & -3 & 7 \end{bmatrix} x = \begin{bmatrix} 4 \\ -6 \\ -3 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 1 & 2 & -1 \\ 0 & -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 1 & 5/3 & -4/3 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 8/3 & 5/3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 1 & 5/3 & -4/3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 1 & 5/3 & -4/3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & 0 & 0 & 0 \\ b & e & 0 & 0 \\ c & f & h & 0 \\ d & g & i & j \end{bmatrix} \begin{bmatrix} 1 & 1 & 5/3 & -4/3 \\ 0 & 1 & 5/3 & -4/3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -2 & 2 \\ 2 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & -3 & 7 \end{bmatrix} \quad \begin{matrix} a=1 & d=2 & g=-1 \\ b=2 & e=3 & h=1/3 \\ c=1 & f=1 & i=8/3 \\ j=-1 \end{matrix}$$

$$-6b + 6e = 30 \quad -6b + 18 = 30 \quad b = -2$$

$$-2b + \frac{5}{3}e = 1 \quad 11e = 33$$

$$-6b + 5e = 3 \quad e = 3$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 1 & 1 & 1/3 & 0 \\ 2 & -1 & 8/3 & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ -3 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 14/3 \\ -9 \\ 4 \end{bmatrix}$$

$$2Y_1 + 3Y_2 = -6$$

$$8 + 3Y_2 = -6$$

$$3Y_2 = -14$$

$$2Y_1 - Y_2 + \frac{8}{3}Y_3 + Y_4 = -10$$

$$4 + \frac{14}{3} - \frac{58}{3} + Y_4 = -10$$

$$Y_1 + Y_2 + \frac{1}{3}Y_3 = -3$$

$$4 - \frac{14}{3} + \frac{1}{3}Y_3 = -3$$

$$\frac{1}{3}Y_3 = \frac{7}{3} \quad Y_3 = 7$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 1 & \frac{5}{3} & -\frac{4}{3} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -\frac{14}{3} \\ 7 \\ 4 \end{bmatrix}$$

$$x_1 + x_2 - 2x_3 + 2x_4 = 4$$

$$x_1 + 9 + 22 + 8 = 4$$

$$x_2 + \frac{5}{3}x_3 - \frac{4}{3}x_4 = -\frac{14}{3}$$

$$A: \begin{bmatrix} -45 \\ 19 \\ -11 \\ 4 \end{bmatrix} \# \checkmark$$

$$x_2 + \frac{5}{3} - 11 - \frac{4}{3}x_4 = -\frac{14}{3}$$

$$= -\frac{55}{3} + \left(-\frac{16}{3}\right)$$

$$= -\frac{71}{3}$$

$$\frac{71}{3} \left(-\frac{14}{3}\right) = x_2$$

$$x_2 = 19$$

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$$(1) \quad 3x_1 - x_2 + x_3 + 5x_4 = 3$$

$$-2x_1 + x_2 - x_3 - 3x_4 = 9$$

$$-2x_1 + x_2 - 7x_4 = 8$$

$$\left[\begin{array}{cccc|c} 3 & -1 & 1 & 5 & 3 \\ -2 & 1 & -1 & -3 & 9 \\ -2 & 1 & 0 & -7 & 8 \end{array} \right] \xrightarrow{\substack{R_2 \leftrightarrow R_1 \\ R_3 \leftrightarrow R_1}} \left[\begin{array}{cccc|c} 3 & -1 & 1 & 5 & 3 \\ 0 & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & 11 \\ 0 & \frac{1}{3} & \frac{2}{3} & -\frac{1}{3} & 10 \end{array} \right] \xrightarrow{\times(3)} \left[\begin{array}{cccc|c} 3 & -1 & 1 & 5 & 3 \\ 0 & 1 & -1 & 1 & 11 \\ 0 & 0 & 1 & -4 & -1 \end{array} \right]$$

$$\xrightarrow{\substack{R_1 \leftrightarrow R_2 \\ R_3 \leftrightarrow R_2}} \left[\begin{array}{cccc|c} 3 & -1 & 1 & 5 & 3 \\ 0 & 1 & -1 & 1 & 11 \\ 0 & 0 & 1 & -4 & -1 \end{array} \right] \xrightarrow{\substack{R_1 \leftrightarrow R_2 \\ R_3 \leftrightarrow R_2}} \left[\begin{array}{cccc|c} 3 & -1 & 1 & 5 & 3 \\ 0 & 1 & 0 & -3 & 32 \\ 0 & 0 & 1 & -4 & -1 \end{array} \right] \xrightarrow{\times(1)} \left[\begin{array}{cccc|c} 3 & 0 & 0 & 6 & 36 \\ 0 & 1 & 0 & -3 & 32 \\ 0 & 0 & 1 & -4 & -1 \end{array} \right]$$

$$\begin{cases} 3x_1 + 6x_4 = 36 \\ x_2 - 3x_4 = 32 \\ x_3 - 4x_4 = -1 \end{cases}$$

$$\Rightarrow x_4 = t \Rightarrow \begin{cases} x_1 = 12 - 2t \\ x_2 = 32 + 3t \\ x_3 = 4t - 1 \\ x_4 = t \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 12 \\ 32 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{aligned} & \xrightarrow{\substack{R_1 \leftrightarrow R_2 \\ R_3 \leftrightarrow R_2}} \left[\begin{array}{cccc|ccc} 1 & 4 & 1 & 1 & 0 & 0 \\ -1 & -3 & -2 & 0 & 1 & 0 \\ 2 & 6 & 6 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{R_1 \leftrightarrow R_2 \\ R_3 \leftrightarrow R_2}} \left[\begin{array}{cccc|ccc} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & -2 & 4 & -2 & 0 & 1 \end{array} \right] \xrightarrow{\substack{R_1 \leftrightarrow R_2 \\ R_3 \leftrightarrow R_2}} \left[\begin{array}{cccc|ccc} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 2 & 1 \end{array} \right] \xrightarrow{\substack{R_1 \leftrightarrow R_2 \\ R_3 \leftrightarrow R_2}} \left[\begin{array}{cccc|ccc} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & \frac{1}{2} \end{array} \right] \\ & \xrightarrow{\substack{R_1 \leftrightarrow R_2 \\ R_3 \leftrightarrow R_2}} \left[\begin{array}{cccc|ccc} 1 & 0 & 5 & -3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 1 & \frac{1}{2} \end{array} \right] \xrightarrow{\substack{R_1 \leftrightarrow R_2 \\ R_3 \leftrightarrow R_2}} \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & -3 & -9 & \frac{5}{2} \\ 0 & 1 & 0 & 1 & 2 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 1 & \frac{1}{2} \end{array} \right] \end{aligned}$$

$$X = \begin{bmatrix} -3 & -9 & \frac{5}{2} \\ 1 & 2 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix}$$

$$-9 \cdot 0 + 8 \cdot 8 + 8 = 6$$

$$9 \cdot 0 - 6 \cdot 6 - 1 \cdot 6 = 9 \cdot 0 - 8 \cdot 2 = 8$$

$$-18 \cdot 0 + 13 \cdot 2 + 4 \cdot 8 = -18 \cdot 0 + 18$$

$$\Rightarrow X = \begin{bmatrix} -95 \\ 23 \\ 9 \end{bmatrix}$$

$$(3) \begin{aligned} 3x_1 + 2x_2 + 0x_3 &= 5 \\ x_1 + 2x_2 - 2x_3 &= 5 \\ 6x_1 + x_2 + 6x_3 &= 5 \end{aligned}$$

$$\Delta \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & -2 \\ 6 & 1 & 6 \end{bmatrix} = (36 - 24 + 1) - (12 - 6 + 12) = -5$$

$$\Delta x_1 \begin{bmatrix} 5 & 2 & 1 \\ 5 & 2 & -2 \\ 5 & 1 & 6 \end{bmatrix} = (60 - 20 + 5) - (10 - 10 + 60) = -15$$

$$\Delta x_2 \begin{bmatrix} 3 & 5 & 1 \\ 1 & 5 & -2 \\ 6 & 5 & 6 \end{bmatrix} = (90 - 60 + 5) - (30 - 30 + 30) = 5$$

$$\Delta x_3 \begin{bmatrix} 3 & 2 & 5 \\ 1 & 2 & 5 \\ 6 & 1 & 5 \end{bmatrix} = (30 + 60 + 5) - (60 + 15 + 10) = 10$$

$$x_1 = \frac{\Delta x_1}{\Delta} = \frac{-15}{-5} = 3$$

$$x_2 = \frac{\Delta x_2}{\Delta} = \frac{5}{-5} = -1$$

$$x_3 = \frac{\Delta x_3}{\Delta} = \frac{10}{-5} = -2$$

$$(x_1, x_2, x_3) = (3, -1, 2)$$

$$(4) \begin{aligned} U &= \begin{bmatrix} 1 & -2 & 2 \\ 2 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & -3 & 7 \end{bmatrix} \xrightarrow{R_2 - R_1, R_3 - R_1, R_4 - R_1} \begin{bmatrix} 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 1 & 2 & -1 \\ 0 & -1 & 1 & 3 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 3 & 5 & -4 \\ 0 & -1 & 1 & 3 \end{bmatrix} \\ &\xrightarrow{R_2 \times \frac{1}{3}, R_3 - 3R_2, R_4 + R_2} \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} \\ 0 & 0 & \frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & \frac{5}{3} & \frac{2}{3} \end{bmatrix} \xrightarrow{R_3 \times 3, R_4 - 5R_3} \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -\frac{1}{3} \end{bmatrix} \\ &\xrightarrow{R_4 \times -3} \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$(2) \begin{bmatrix} 1 & -2 & 2 \\ 2 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & -3 & 7 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & 0 \\ b & e & 0 & 0 \\ c & f & h & 0 \\ d & g & i & j \end{bmatrix} \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$4 - 14 - 42 +$$

$$8 + 3y_2 = -6$$

$$3y_2 = -14$$

$$y_2 = -\frac{14}{3}$$

$$\frac{1}{3}y_3 - 10 = -3$$

$$\frac{1}{3}y_3 = 7$$

$$y_3 = 21$$

$$4 - \frac{14}{3} + \frac{1}{3}y_3 = -3$$

$$\frac{1}{3}y_3 = -7 - \frac{14}{3} = -\frac{35}{3}$$

$$y_3 = -35$$

$$8 + 14 + 56 - y_4 = -10$$

$$y_4 = 88$$

$$\begin{cases} y_1 = 4 \\ y_2 = -\frac{14}{3} \\ y_3 = -35 \\ y_4 = 88 \end{cases}$$

$$8 + \frac{14}{3} - \frac{280}{3} - y_4 = -10$$

$$y_4 = 18 - \frac{266}{3} = \frac{54 - 266}{3}$$

$$= \frac{-212}{3}$$

$$L: \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 1 & 1 & \frac{1}{3} & 0 \\ 2 & -1 & \frac{8}{3} & -1 \end{bmatrix}$$

$$A = L U$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 1 & 1 & \frac{1}{3} & 0 \\ 2 & -1 & \frac{8}{3} & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ -3 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -\frac{14}{3} \\ -7 \\ 4 \end{bmatrix}$$

$$\begin{cases} x_1 = -45 \\ x_2 = 19 \\ x_3 = -11 \\ x_4 = 4 \end{cases} \quad (x_1, x_2, x_3, x_4) = (-45, 19, -11, 4)$$

$$1 = a + 0 + 0 + 0, a = 1$$

$$2 = b + 0 + 0 + 0, b = 2$$

$$1 = c + 0 + 0 + 0, c = 1$$

$$2 = d + 0 + 0 + 0, d = 2$$

$$5 = b + e + 0 + 0, e = 3$$

$$2 = c + f + 0 + 0, f = 1$$

$$1 = d + g + 0 + 0, g = -1$$

$$0 = -2c + \frac{5}{3}f + h + 0, h = 2 - \frac{5}{3} = \frac{1}{3}$$

$$-3 = -2d + \frac{5}{3}g + i, i = -3 + 4 + \frac{5}{3} = \frac{8}{3}$$

$$7 = 2d - \frac{4}{3}g + i + j, j = 7 - 4 - \frac{4}{3} - \frac{8}{3} = 3 - \frac{12}{3} = -1$$

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高斯

$$\begin{aligned} 3x_1 - x_2 + x_3 + 5x_4 &= 3 \\ -2x_1 + x_2 - x_3 - 3x_4 &= 9 \\ -2x_1 + x_2 + 0 - 7x_4 &= 8 \end{aligned}$$

$$\begin{bmatrix} 3 & -1 & 1 & 5 & 3 \\ -2 & 1 & -1 & -3 & 9 \\ -2 & 1 & 0 & -7 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{5}{3} & 1 \\ 0 & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 11 \\ 0 & \frac{1}{3} & \frac{2}{3} & -\frac{11}{3} & 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{5}{3} & 1 \\ 0 & 1 & -1 & 1 & 33 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{3} & 0 & 3 & \frac{4}{3} \\ 0 & 1 & 0 & -3 & 32 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 12 \\ 0 & 1 & 0 & -3 & 32 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

令 $x_4 = t$.

$$x_1 = 12 - 2t$$

$$x_2 = 32 + 3t$$

$$x_3 = -1 + 4t$$

無限多解

反矩陣

$$\begin{bmatrix} 1 & 4 & 1 \\ -1 & -3 & -2 \\ 2 & 6 & 6 \end{bmatrix} X = \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 1 & | & 1 & 0 & 0 \\ -1 & -3 & -2 & | & 0 & 1 & 0 \\ 2 & 6 & 6 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & 1 & 1 & 0 \\ 0 & -2 & 4 & | & -2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & 1 & 1 & 0 \\ 0 & 0 & 2 & | & 0 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & 1 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 1 & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 0 & | & 1 & -1 & -\frac{1}{2} \\ 0 & 1 & 0 & | & 1 & 2 & \frac{1}{2} \\ 0 & 0 & 1 & | & 0 & 1 & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & -3 & -9 & -\frac{5}{2} \\ 0 & 1 & 0 & | & 1 & 2 & \frac{1}{2} \\ 0 & 0 & 1 & | & 0 & 1 & \frac{1}{2} \end{bmatrix}$$

$$X = \begin{bmatrix} -3 & -9 & -\frac{5}{2} \\ 1 & 2 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix} = \begin{bmatrix} -95 \\ 23 \\ 9 \end{bmatrix}$$

3. 克拉理

$$3x_1 + 2x_2 + x_3 = 5$$

$$x_1 + 2x_2 - 2x_3 = 5$$

$$6x_1 + x_2 + 6x_3 = 5$$

$$x_1 = \frac{\begin{vmatrix} 5 & 2 & 1 \\ 5 & 2 & -2 \\ 5 & 1 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & -2 \\ 6 & 1 & 6 \end{vmatrix}}$$

$$\begin{aligned} & (60 + (-20) + 5) - (12 - 12 + 60) \\ & = -15 \end{aligned}$$

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & -2 \\ 6 & 1 & 6 \end{vmatrix}$$

$$\begin{aligned} & (36 - 24 + 1) - (12 - 6 + 12) \\ & = -5 \end{aligned}$$

$$= 3$$

$$x_2 = \frac{\begin{vmatrix} 5 & 1 & 3 \\ 5 & -2 & 1 \\ 5 & 6 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & -2 \\ 6 & 1 & 6 \end{vmatrix}}$$

$$\begin{aligned} & (90 - 60 + 15) - (30 - 30 + 30) \\ & = 5 \end{aligned}$$

$$x_1 = 3$$

$$x_2 = -1$$

$$x_3 = -2$$

(3)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & \frac{1}{3} & 1 & 0 \\ 2 & -\frac{1}{3} & 8 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ -3 \\ -10 \end{bmatrix}$$

$$x_1 = 4$$

$$2x_1 + x_2 = -6 \quad x_2 = -14$$

$$x_1 + \frac{1}{3}x_2 + x_3 = -3 \quad x_3 = -\frac{1}{3}$$

$$2x_1 - \frac{1}{3}x_2 + 8x_3 + x_4 = -10 \quad x_4 = -4$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -14 \\ -\frac{1}{3} \\ -4 \end{bmatrix}$$

$$x_1 + x_2 - 2x_3 + 2x_4 = 4 \quad x_1 = -45$$

$$3x_2 + 5x_3 - 4x_4 = -14 \quad x_2 = 19$$

$$\frac{1}{3}x_3 + \frac{1}{3}x_4 = -\frac{1}{3} \quad x_3 + x_4 = -1$$

$$x_4 = 4$$

$$x_3 = -1$$

$$4. \begin{bmatrix} 1 & 1 & -2 & 2 \\ 2 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & -3 & 7 \end{bmatrix} x = \begin{bmatrix} 4 \\ -6 \\ -3 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 1 & 2 & -1 \\ 0 & -1 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a & 0 & 0 & 0 \\ b & c & 0 & 0 \\ c & f & h & 0 \\ d & g & i & j \end{bmatrix} \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -2 & 2 \\ 2 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & -3 & 7 \end{bmatrix}$$

$$x = \begin{bmatrix} -45 \\ 19 \\ -1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{8}{3} & \frac{5}{3} \end{bmatrix}$$

$$a = 1$$

$$b + 3e = 5$$

$$-2c + 5f + \frac{1}{3}h = 0$$

$$b = 2$$

$$e = 1$$

$$h = 1$$

$$c = 1$$

$$c + 3f = 2$$

$$-2d + 5g + \frac{1}{3}i = -3$$

$$f = \frac{1}{3}$$

$$i = 8$$

$$d + 3g = 1$$

$$2d - 4g + \frac{1}{3}i - j = 7$$

$$g = -\frac{1}{3}$$

$$j = 1$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & \frac{1}{3} & 1 & 0 \\ 2 & -\frac{1}{3} & 8 & 1 \end{bmatrix}$$

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$$(1) \begin{cases} 3x_1 - x_2 + x_3 + 5x_4 = 3 \\ -2x_1 + x_2 - x_3 - 3x_4 = 9 \\ -2x_1 + x_2 - 7x_4 = 8 \end{cases}$$

$$\begin{bmatrix} 3 & -1 & 1 & 5 & 3 \\ -2 & 1 & -1 & -3 & 9 \\ -2 & 1 & 0 & -7 & 8 \end{bmatrix} \xrightarrow{2 \times -1} \begin{bmatrix} 3 & -1 & 1 & 5 & 3 \\ -2 & 1 & -1 & -3 & 9 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

$$\xrightarrow{+2} \begin{bmatrix} 1 & 0 & 0 & 2 & 12 \\ -2 & 1 & -1 & -3 & 9 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix} \xrightarrow{+2} \begin{bmatrix} 1 & 0 & 0 & 2 & 12 \\ 0 & 1 & -1 & -4 & 33 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

$$\xrightarrow{+1} \begin{bmatrix} 1 & 0 & 0 & 2 & 12 \\ 0 & 1 & 0 & -3 & 32 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 12 \\ 0 & 1 & 0 & -3 & 32 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix} \xrightarrow{\times 1} \begin{bmatrix} 1 & 0 & 0 & 2 & 12 \\ 0 & 1 & 0 & -3 & 32 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

$$\begin{cases} x_1 + 2x_4 = 12 \\ x_2 - 3x_4 = 32 \\ x_3 - 4x_4 = -1 \end{cases}$$

$$\frac{1}{2} x_4 = t$$

$$\begin{cases} x_1 = 12 - 2t \\ x_2 = 32 - 3t \\ x_3 = -1 + 4t \end{cases}$$

$$(2) \begin{bmatrix} 1 & 4 & 1 \\ -1 & -3 & -2 \\ 2 & 6 & 6 \end{bmatrix} x = \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 4 & 1 \\ -1 & -3 & -2 \\ 2 & 6 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 1 \\ -1 & -3 & -2 \\ 2 & 6 & 6 \end{vmatrix}$$

$$= -18 - 16 - 6 + 6 + 12 + 24 = 2 \neq 0 \text{ 有 } A^{-1}$$

$$A^{-1} = \begin{bmatrix} 1 & 4 & 1 & 1 & 0 & 0 \\ -1 & -3 & -2 & 0 & 1 & 0 \\ 2 & 6 & 6 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\times (-2)} \begin{bmatrix} 1 & 4 & 1 & 1 & 0 & 0 \\ -1 & -3 & -2 & 0 & 1 & 0 \\ 2 & 6 & 6 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{\times -4} \begin{bmatrix} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & -2 & 4 & -2 & 0 & 1 \end{bmatrix} \xrightarrow{\times 2} \begin{bmatrix} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & -2 & 4 & -2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 5 & -3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 2 & 6 & 2 & 1 \end{bmatrix}$$

$$\xrightarrow{\times -5} \begin{bmatrix} 1 & 0 & 5 & -3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 6 & 1 & 1/2 \end{bmatrix} \xrightarrow{\times 1} \begin{bmatrix} 1 & 0 & 5 & -3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 6 & 1 & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -3 & -9 & -5/2 \\ 0 & 1 & 0 & 1 & 2 & 1/2 \\ 0 & 0 & 1 & 6 & 1 & 1/2 \end{bmatrix} \begin{bmatrix} -3 & -9 & -5/2 \\ 1 & 2 & 1/2 \\ 0 & 1 & 1/2 \end{bmatrix} = A^{-1}$$

$$x = \begin{bmatrix} -3 & -9 & -5/2 \\ 1 & 2 & 1/2 \\ 0 & 1 & 1/2 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix} = \begin{bmatrix} -95 \\ 23 \\ 9 \end{bmatrix}$$

$$(-3 \times 6) + (-9 \times 8) + (-5/2 \times 2) = -18 - 72 - 5 = -95$$

$$(1 \times 6) + (2 \times 8) + (1/2 \times 2) = 6 + 16 + 1 = 23$$

$$(0 \times 6) + (1 \times 8) + (1/2 \times 2) = 0 + 8 + 1 = 9$$

$$\begin{cases} 3x_1 + 2x_2 + x_3 = 5 \\ x_1 + 2x_2 - 2x_3 = 5 \\ 6x_1 + x_2 + 6x_3 = 5 \end{cases}$$

$$x_1 = \frac{\begin{vmatrix} 5 & 2 & 1 \\ 5 & 2 & -2 \\ 5 & 1 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & -2 \\ 6 & 1 & 6 \end{vmatrix}} = \frac{-15}{-5} = 3$$

$$x_2 = \frac{\begin{vmatrix} 3 & 5 & 1 \\ 1 & 5 & -2 \\ 6 & 5 & 6 \end{vmatrix}}{-5} = \frac{5}{-5} = -1$$

$$x_3 = \frac{\begin{vmatrix} 3 & 2 & 5 \\ 1 & 2 & 5 \\ 6 & 1 & 5 \end{vmatrix}}{-5} = \frac{10}{-5} = -2$$

Ans: $x_1 = 3, x_2 = -1, x_3 = -2$ ✓

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & -2 \\ 6 & 1 & 6 \end{vmatrix} \begin{vmatrix} 3 & 2 \\ 1 & 2 \\ 6 & 1 \end{vmatrix} = 3 \cdot 6 - 24 + 1 \cdot 12 + 6 - 12 = -5$$

$$\begin{vmatrix} 5 & 2 & 1 \\ 5 & 2 & -2 \\ 5 & 1 & 6 \end{vmatrix} \begin{vmatrix} 5 & 2 \\ 5 & 2 \\ 5 & 1 \end{vmatrix} = 60 - 20 + 5 - 10 + 10 - 60 = -15$$

$$\begin{vmatrix} 3 & 5 & 1 \\ 1 & 5 & -2 \\ 6 & 5 & 6 \end{vmatrix} \begin{vmatrix} 3 & 5 \\ 1 & 5 \\ 6 & 5 \end{vmatrix} = 90 - 60 + 5 - 30 + 30 - 30 = 5$$

$$\begin{vmatrix} 3 & 2 & 5 \\ 1 & 2 & 5 \\ 6 & 1 & 5 \end{vmatrix} \begin{vmatrix} 3 & 2 \\ 1 & 2 \\ 6 & 1 \end{vmatrix} = 30 + 60 + 5 - 60 - 15 - 10 = 10$$

(4) LU分解法

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 1 & -3 \end{bmatrix} x = \begin{bmatrix} 4 \\ -6 \\ -3 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 1 & -3 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & 0 \\ b & c & 0 & 0 \\ c & f & h & 0 \\ d & g & i & j \end{bmatrix} \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 0 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 0 & 1/3 & 1/3 \\ 0 & 0 & 8/3 & 5/3 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 1/3 & 1 & 0 \\ 2 & -1/3 & 8 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 0 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -14 \\ -1/3 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 0 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = U$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1/3 & 0 & 0 \\ 1 & 1/3 & 1 & 0 \\ 2 & -1/3 & 8 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ -3 \\ -10 \end{bmatrix}$$

$$\begin{cases} y_1 = 4 \\ 2y_1 + y_2 = -6 \\ y_1 + \frac{1}{3}y_2 + y_3 = -3 \\ 2y_1 - \frac{1}{3}y_2 + 8y_3 + y_4 = -10 \end{cases}$$

$$y_1 = 4, y_2 = -14, y_3 = -1/3, y_4 = -4$$

$$\begin{cases} x_4 = 4 \\ \frac{1}{3}x_3 + \frac{1}{3}x_4 = -1/3 \\ 3x_2 + 5x_3 - 4x_4 = -14 \\ x_1 + x_2 - 2x_3 + 2x_4 = 4 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -14 \\ -1/3 \\ 4 \end{bmatrix}$$

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$$\begin{aligned}
 (1) \quad & 3x_1 - x_2 + x_3 + 5x_4 = 3 \\
 & -2x_1 + x_2 - x_3 - 3x_4 = 9 \\
 & -2x_1 + x_2 - 7x_4 = 8
 \end{aligned}$$

$$\begin{bmatrix} 3 & -1 & 1 & 5 & 3 \\ -2 & 1 & -1 & -3 & 9 \\ -2 & 1 & 0 & -7 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 12 \\ -2 & 1 & -1 & -3 & 9 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 12 \\ 0 & 1 & -1 & -3 & 9 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 12 \\ 0 & 1 & 0 & -3 & 32 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

$x_1 \ x_2 \ x_3 \ x_4$

$$\frac{1}{2}x_4 = t$$

$$\begin{aligned}
 x_1 + 2t &= 12 \\
 x_1 &= 12 - 2t
 \end{aligned}$$

$$\begin{aligned}
 x_2 - 3t &= 32 \\
 x_2 &= 32 + 3t
 \end{aligned}$$

$$\begin{aligned}
 x_3 - 4t &= -1 \\
 x_3 &= -1 + 4t
 \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 12 - 2t \\ 32 + 3t \\ -1 + 4t \\ t \end{bmatrix}$$

$$= \begin{bmatrix} 12 \\ 32 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 4 & 1 \\ -1 & -3 & -2 \\ 2 & 6 & 6 \end{bmatrix}$$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 4 & 1 & 1 & 0 & 0 \\ -1 & -3 & -2 & 0 & 1 & 0 \\ 2 & 6 & 6 & 0 & 0 & 1 \end{array} \right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 2 & 6 & 6 & 0 & 0 & 1 \end{array} \right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & -2 & 4 & -2 & 0 & 1 \end{array} \right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 2 & 1 \end{array} \right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 5 & -3 & -4 & -5 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1/2 \end{array} \right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & -9 & -5/2 \\ 0 & 1 & 0 & 1 & 2 & 1/2 \\ 0 & 0 & 1 & 0 & 1 & 1/2 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -3 & -9 & -5/2 \\ 1 & 2 & 1/2 \\ 0 & 1 & 1/2 \end{bmatrix}$$

$$X = A^{-1}b$$

$$= \begin{bmatrix} -3 & -9 & -5/2 \\ 1 & 2 & 1/2 \\ 0 & 1 & 1/2 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix}$$

$$= 6 \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} + 8 \begin{bmatrix} -9 \\ 2 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -5/2 \\ 1/2 \\ 1/2 \end{bmatrix}$$

$$= \begin{bmatrix} -18 - 72 - 5 \\ 6 + 16 + 1 \\ 0 + 8 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} -95 \\ 23 \\ 9 \end{bmatrix}$$

3.

$$\begin{cases} 3x_1 + 2x_2 + x_3 = 5 \\ x_1 + 2x_2 - 2x_3 = 5 \\ 6x_1 + x_2 + 6x_3 = 5 \end{cases}$$

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & -2 \\ 6 & 1 & 6 \end{vmatrix}$$

$$\det(A) = 3 \cdot 6 + (-2 \cdot 4) + 1 \cdot 12 + 6 \cdot 12$$

$$= 13 - 18$$

$$= -5$$

有唯一解

$$x_1 = \begin{vmatrix} 5 & 2 & 1 \\ 5 & 2 & -2 \\ 5 & 1 & 6 \end{vmatrix}$$

$$5 \cdot 0 + (-20) + 5 \cdot 10 = -10 - 60$$

$$= -15$$

$$x_2 = \begin{vmatrix} 3 & 5 & 1 \\ 1 & 5 & -2 \\ 6 & 5 & 6 \end{vmatrix}$$

$$3 \cdot 0 + (-60) + 5 \cdot 20 - (-30) - 30$$

$$= 5$$

$$x_3 = \begin{vmatrix} 3 & 2 & 5 \\ 1 & 2 & -5 \\ 6 & 1 & 5 \end{vmatrix}$$

$$= 30 + 60 + 5 - 60 - 15 - 10$$

$$= 10$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$$

4.

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 2 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & -3 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 5 & 3 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{8}{3} & \frac{5}{3} \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 2 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & -3 & 1 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & 0 \\ b & e & 0 & 0 \\ c & f & h & 0 \\ d & g & i & j \end{bmatrix} \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$a = 1$$

$$b = 2, b + 3e = 5, e = 1$$

$$c = 1, c + 3f = 2, f = \frac{1}{3}$$

$$d = 2, d + 3g = 1, g = -\frac{1}{3}$$

$$-2c + 5f + \frac{1}{3}h = 0$$

$$-\frac{2}{3} + \frac{5}{3} + \frac{1}{3}h = 0$$

$$h = 1$$

$$-2d + 5g + \frac{1}{3}i = -3$$

$$-4 - \frac{5}{3} + \frac{1}{3}i = -3$$

$$-12 + 5 + i = -9$$

$$i = 8$$

$$2d - 4g + \frac{1}{3}i - j = 9$$

$$6d - 12g + i - 3j = 21$$

$$12 + 4 + 8 - 3j = 21$$

$$3 = 3j$$

$$j = 1$$

$$Ux = Y$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -14 \\ -\frac{11}{3} \\ -4 \end{bmatrix}$$

$$x_4 = 4$$

$$\frac{1}{3}x_3 + \frac{1}{3}x_4 = -\frac{11}{3}, x_3 = -11$$

$$3x_2 + 5x_3 - 4x_4 = -14, x_2 = 19$$

$$x_1 + x_2 - 2x_3 + 2x_4 = 4, x_1 = -45$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -45 \\ 19 \\ -11 \\ 4 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & \frac{1}{3} & 1 & 0 \\ 2 & -\frac{1}{3} & 8 & 1 \end{bmatrix}$$

$$\frac{A}{LU} x = b, LY = b$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & \frac{1}{3} & 1 & 0 \\ 2 & -\frac{1}{3} & 8 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ -3 \\ -10 \end{bmatrix}$$

$$y_1 = 4$$

$$2y_1 + y_2 = -6, y_2 = -14$$

$$y_1 + \frac{1}{3}y_2 + y_3 = -3, y_3 = -\frac{2}{3}$$

$$2y_1 - \frac{1}{3}y_2 + 8y_3 + y_4 = -10, y_4 = -4$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -14 \\ -\frac{2}{3} \\ -4 \end{bmatrix}$$

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$$1. \begin{bmatrix} 3 & -1 & 1 & 5 & 3 \\ -2 & 1 & 1 & -3 & 9 \\ -2 & 1 & 0 & -7 & 8 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{5}{3} & 1 \\ 0 & 1 & -1 & 1 & 33 \\ 0 & 1 & -2 & -11 & 30 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{5}{3} & 1 \\ 0 & 1 & -1 & 1 & 33 \\ 0 & 0 & -3 & 12 & 3 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{5}{3} & 1 \\ 0 & 1 & -1 & 1 & 33 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{5}{3} & 1 \\ 0 & 1 & 0 & -\frac{4}{3} & -\frac{32}{3} \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{35}{3} \\ 0 & 1 & 0 & -3 & 32 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 2 & 12 \\ 0 & 1 & 0 & -3 & 32 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

$$x_1 + 2x_4 = 12$$

$$\text{令 } x_4 = t$$

$$x_2 - 3x_4 = 32$$

$$x_1 = 12 - 2t$$

$$x_3 - 4x_4 = -1$$

$$x_2 = 32 + 3t$$

$$x_3 = -1 + 4t \quad x_4 = t$$

$$2. \begin{bmatrix} 1 & 4 & 1 & 1 & 0 & 0 \\ -1 & -3 & -2 & 0 & 1 & 0 \\ 2 & 6 & 6 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 2 & -4 & 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 1 & -2 & -1 & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 1 & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & -3 & -8 & -2 \\ 0 & 1 & 0 & 1 & 2 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -3 & -9 & -\frac{5}{2} \\ 0 & 1 & 0 & 1 & 2 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 1 & \frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} -3 & -9 & -\frac{5}{2} \\ 1 & 2 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix} = \begin{bmatrix} -95 \\ 23 \\ 9 \end{bmatrix}$$

3.

$$\det(A) = \begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & -2 \\ 6 & 1 & 6 \end{vmatrix} = 36 + (-24) + 1 - (12 + (-6) + 12) = -5$$

$$x_1 = \frac{\begin{vmatrix} 5 & 2 & 1 \\ 5 & 2 & -2 \\ 5 & 1 & 6 \end{vmatrix}}{-5}$$

$$x_2 = \frac{\begin{vmatrix} 3 & 5 & 1 \\ 1 & 5 & -2 \\ 6 & 5 & 6 \end{vmatrix}}{-5}$$

$$x_3 = \frac{\begin{vmatrix} 3 & 2 & 5 \\ 1 & 2 & 5 \\ 6 & 1 & 5 \end{vmatrix}}{-5}$$

$$= \frac{-15}{-5}$$

$$= \frac{5}{-5}$$

$$= \frac{10}{-5}$$

$$= 3$$

$$= -1$$

$$= -2$$

4.

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 2 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & -3 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 3 & 5 & -4 \\ 0 & 1 & 2 & -1 \\ 0 & 1 & -1 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 1 & \frac{5}{3} & -\frac{4}{3} \\ 0 & 1 & 2 & -1 \\ 0 & 1 & -1 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 1 & \frac{5}{3} & -\frac{4}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{8}{3} & \frac{5}{3} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 1 & \frac{5}{3} & -\frac{4}{3} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & \frac{5}{3} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 1 & \frac{5}{3} & -\frac{4}{3} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 2 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & -3 & 7 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & 0 \\ b & e & 0 & 0 \\ c & f & h & 0 \\ d & g & i & j \end{bmatrix} \begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 1 & \frac{5}{3} & -\frac{4}{3} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow$$

$$a = 1 \quad 5 = b + e, \quad e = 3$$

$$b = 2 \quad 2 = c + f, \quad f = 1$$

$$c = 1 \quad 1 = d + g, \quad g = -1$$

$$d = 2$$

$$0 = -2c + \frac{5}{3}f + h, \quad h = \frac{1}{3}$$

$$-3 = -2d + \frac{5}{3}g + i, \quad i = \frac{8}{3}$$

$$7 = 2d - \frac{4}{3}g + i + j, \quad j = -1$$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 1 & 1 & \frac{1}{3} & 0 \\ 2 & -1 & \frac{8}{3} & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 1 & 1 & \frac{1}{3} & 0 \\ 2 & -1 & \frac{8}{3} & -1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ -3 \\ -10 \end{bmatrix} \Rightarrow$$

$$d_1 = 4$$

$$2d_1 + 3d_2 = -6, \quad d_2 = \frac{-14}{3}$$

$$d_1 + d_2 + \frac{1}{3}d_3 = -3, \quad d_3 = -7$$

$$2d_1 - d_2 + \frac{8}{3}d_3 + d_4 = -10, \quad d_4 = 4$$

$$\begin{bmatrix} 1 & 1 & -2 & 2 \\ 0 & 1 & \frac{5}{3} & -\frac{4}{3} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ \frac{-14}{3} \\ -7 \\ 4 \end{bmatrix}$$

$$4 = x_1 + x_2 - 2x_3 + 2x_4, \quad x_1 = 53$$

$$\frac{-14}{3} = x_2 + \frac{5}{3}x_3 - \frac{4}{3}x_4, \quad x_2 = \frac{57}{3} = 19$$

$$-7 = x_3 + x_4, \quad x_3 = -11$$

$$4 = x_4$$

$$\therefore x = \begin{bmatrix} -45 \\ 19 \\ -11 \\ 4 \end{bmatrix} \checkmark$$